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The role of transition in determining friction and heat transfer in smooth and rough passages

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Abstract—Transition can have profound effects on friction and heat transfer in flow passages. According to the proposed law of corresponding states for friction, different types of roughness elements exhibit the same behavior for friction at the same reduced conditions, as is the case for smooth passages of arbitrary shapes. For smooth or rough tubes, it appears that the average heat transfer coefficient decreases with increasing critical Reynolds number for transition. For non-circular smooth passages, average heat transfer coefficients can be calculated from the circular tube relations, provided the critical values for friction factor and Reynolds number are known.

1. INTRODUCTION

TRANSITION, a process whereby the motion of a fluid changes from laminar to turbulent flow, was successfully traced by Reynolds [1] over a century ago. Although this remarkable process occurs naturally when the critical velocity and pressure attain some threshold values, it is also well known that its onset can be delayed or accelerated markedly. For any particular condition, be it of a natural origin or one in which the process is effected by artificial means, the most notable characteristic of transition is the marked change in the resistance to flow. Despite the extensive studies on transition [2, 3], and even though it is of considerable importance in convective heat transfer, the underlying physics and the implications of this phenomenon have eluded complete understanding.

The connection between fluid friction and heat transfer was also postulated by Reynolds [4] and the resulting relation is generally referred to as the Reynolds analogy. Thereafter, the problem of friction and heat transfer analogy was studied by very eminent researchers and an overview of the many contributions on the subject is given by Colburn [5]. Rather surprisingly, although there are clear indications that transition can have very profound effects on surface friction, it does not appear that hitherto any systematic investigations have been made to establish the connections between transition and heat transfer. Consequently, the origin of the differences among the various results obtained on smooth passages, or of the anomaly between the magnitudes of the increase in frictional pressure coefficients and heat transfer coefficients in the presence of roughness, has been open to conjecture.

The work described in this paper has its origin in a re-analysis of the literature on friction in smooth circular and non-circular passages which resulted in the formulation of the critical friction method or the frictional law of corresponding states, as well as the fundamental requirements for flow similarity [6]. However, it became quite evident that the implications of transition to friction or heat transfer are far too important and more numerous than can be inferred from much that has been written on the subject. It was with this understanding that the aforementioned study was extended to include friction and heat transfer in smooth and rough passages. As with the earlier study, this analysis was carried out using existing literature data, the complete tabulation of which is given in ref. [7].

2. FRICTION

2.1. *Smooth passages*

The critical friction law (or the law of corresponding states) asserts that flows having the same critical Reynolds number and critical friction factor are dynamically similar to one another [6]. This law was successfully applied to smooth tubes of arbitrary shapes. The simple mathematical relations that are needed to effect such an analysis are given by equations (1) and (2)

$$\psi_R = Re_{c,r}/Re_{c,a}; \quad \psi_f = f_{c,r}/f_{c,a} \tag{1}$$

NOMENCLATURE

a_r	aspect ratio, w/s
b	base of triangular duct [m]
D, D_e	diameter, equivalent (hydraulic) diameter [m]
d_i, d_o	inner, outer diameter of annular space [m]
e	roughness height [m]
f	Fanning friction factor
f_m	reduced friction factor, equation (2)
f_a	friction factor for arbitrary condition, equation (2)
h	height of triangular duct [m]
$\bar{h}$	heat transfer coefficient [W m^{-2} °C^{-1}]
k	fluid thermal conductivity [W m^{-1} °C^{-1}]
L_e	entrance length [m]
Nu	Nusselt number, $\bar{h}D_e/k$
Nu_m	Nusselt number associated with Re_m
Pr	Prandtl number, ν/α
p	roughness pitch [m]
Re	Reynolds number
Re_m	similarity parameter or reduced Reynolds number, equation (2)
Re_a	Reynolds number for arbitrary condition, equation (2)
s	channel spacing [m]
w	channel width [m].

Greek symbols

α	apex angle [deg]; thermal diffusivity [m^2 s^{-1}]
β	diameter ratio, d_i/d_o
μ	fluid viscosity [Pa s]
ν	kinematic viscosity [m^2 s^{-1}]
ϕ_f	friction factor ratio, $f_m/f_{m,s}$
ϕ_h	ratio of heat transfer coefficient, Nu_r/Nu_s
ψ_f	ratio of critical f, equation (1)
ψ_R	ratio of critical Re, equation (1).

Additional subscripts

a	arbitrary condition
c,r	critical parameters for reference condition
c,a	critical parameters for arbitrary condition
max	maximum value
m,s ;s	smooth duct.

$$Re_m = \psi_R \, Re_a; \quad f_m = \psi_f f_a. \qquad (2)$$

In equation (1), $R_{c,r}$ and $f_{c,r}$ are the reference values for the critical Reynolds number and critical friction factor, while the corresponding critical parameters for an arbitrary passage are $Re_{c,a}$ and $f_{c,a}$. In equation (2), the similarity parameters Re_m and f_m are independent of the length scale used to reduce the data to non-dimensional form. For rectangular or annular passages, and with $Re_{c,r} = 2100$ as the reference, ψ_R decreases with increasing aspect or diameter ratio. For triangular passages, ψ_R increases with increasing height-to-base ratio or with decreasing apex angle in the case of isosceles channels. For fully developed flow in smooth passages, the limiting critical (Fanning) friction factor is about 0.008, provided that the data are reduced using the hydraulic (or equivalent) diameter, D_e.

To shed further light on the application of the law of corresponding states to flow passages, data for rectangular, annular and triangular passages (a total of 500 points) that were presented as three separate figures in the aforementioned paper have been consolidated into a single plot (Fig. 1). The well-known laminar and turbulent relations, as well as the circular tube data of Dodge and Metzner [8], are also included in Fig. 1 for purposes of comparison. The number of data points for each cross-section is given in parentheses in Fig. 1. Given the differences in the relative entrance length (L_e/D_e), the relative roughness of the smooth ducts, and the lack of geometrically similar test conditions, all of which affect the critical Reynolds number, the results in Fig. 1 are satisfactorily consistent.

2.2. *Rough passages*

It is most instructive to begin this analysis by considering the effects of roughness on the critical parameters for transition, and the relevant data are summarized in Table 1. To gain more insight into the influence of roughness on transition, the results of Koch [10] are reproduced exactly in Fig. 2. For these results, the pitch-to-height ratio (p/e) was held fixed at 9.8, hence the figure shows the generally expected

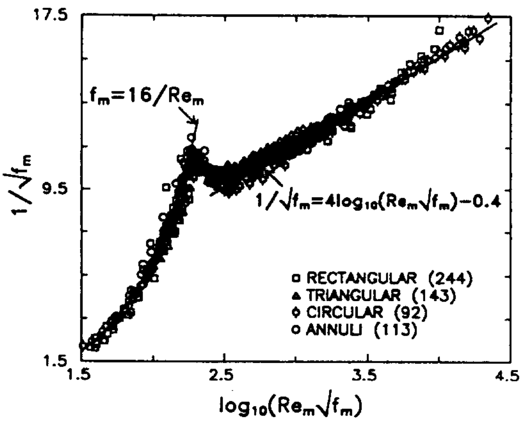

FIG. 1. Friction factor for circular and non-circular passages.

Table 1. Summary of critical data for rough passages

Author(s)	L_e/D	$f_{c,a}\times 10^2$	$Re_{c,a}$	p/e	e/D	Roughness geometry
Nunner [9]		0.83[1]	2510	—	—	(smooth tube)
(fluid—air)		1.31	1908[2]	20.0	0.04	transverse rectangular ring inserts
		1.31	1908[2]	20.0	0.04	
		0.95	2537	80.0	0.04	transverse semi-circular ring
	50	1.14	2497	2.0	0.08	inserts with no clearance at the
		1.65	2014	5.0	0.08	wall
		1.92	1653	10.0	0.08	
		3.36	1022	20.0	0.08	
Koch [10]		4.62	1019	3.92	0.1	
(fluid—air)		3.68	972	9.8	0.1	transverse disc-shaped inserts
		3.72	747	65.0	0.1	with no clearance at the wall
		47.33	226	3.92	0.2	
	50	28.88	244	9.8	0.2	
		57.86	181	26.0	0.25	
		3.16	1170	9.8	0.045	transverse ring inserts with
		7.17	702	9.8	0.075	clearance for flow near the wall
		0.80	2944 (2900)[3]	—	—	(smooth tube)
Marner and Bergles [11] (fluid—ethylene glycol)	—	3.11	1133	6.0[4]	0.088[4]	forge-fin tubes
Knudsen and Katz [12][5]	30.9	1.3	1766	—	—	(smooth)
(fluid—water)	33.0	1.5	1502	0.952	0.089	
	42.1	3.0	784	1.15	0.184	helically-finned annular tubes
	41.1	2.1	1093	0.513	0.185	
	50.1	3.9	605	0.730	0.236	
	52.9	3.5	666	0.452	0.250	
	54.3	4.4	536	0.299	0.269	
Watkinson *et al.* [13]	29.9	3.10	938	10.0	0.097 (1)[6]	
(fluid—oil)	35.6	2.77	756	13.0	0.105 (2)	
	43.6	3.21	989	20.0	0.118 (3)	
	51.6	3.18	949	14.5	0.157 (4)	high spiral fins (HSF)
	57.1	2.83	693	33.3	0.144 (5)	
	24.0	3.44	886	6.0	0.088 (9)	
	20.1	11.35	731	6.6	0.149 (18)	
	33.8	1.65	1464	—	0.099 (11)	
	52.9	2.18	1299	—	0.148 (13)	
	42.9	2.63	1112	—	0.111 (4)	high straight fins (STF)
	52.5	1.97	1290	—	0.094 (16)	
	24.1	2.53	1483	—	0.073 (20)	
	23.8	2.37	2010	162.2	0.037 (15)	
	30.5	1.81	1296	206.9	0.037 (17)	
	19.4	3.71	2127	242.0	0.026 (19)	low spiral fins (LSF)
	24.6	1.93	1482	164.6	0.042 (21)	
	41.5	1.49	1579	444.4	0.047 (22)	
	41.3	1.54	1478	181.8	0.057 (23)	

[1] The conventional definition of friction factor was not adopted in this study.
[2] Characterized by the same profile in the laminar and transition regions.
[3] Smooth tube critical value reported in the original study.
[4] Values deduced from Watkinson *et al.* [13] for Tube #9.
[5] The characteristic dimension is the hydraulic diameter, $D_e = d_o - d_i$.
[6] Numbers in parentheses correspond to tube numbers in the original paper.

trend with increasing height of roughness. It should be emphasized that the data of Table 1 as well as those on Fig. 2 were obtained in the absence of heat transfer (isothermal conditions). Also, to provide a consistent basis for viewing of the critical parameters, the characteristic length for the results in Table 1 is the maximum internal tube diameter. The only exception occurs with the results of Knudsen and Katz [12] which, due to the use of annular passages, have the hydraulic diameter as the linear parameter.

From a comparison of the results (Table 1) of Nunner [9] and Koch [10], both of which were obtained using the same experimental facility, it may be noted that Nunner's $f_{c,a}$ data are much lower, while the values for $Re_{c,a}$ are substantially higher, than those determined from Koch's results. These cannot be attributed solely to differences in the details of roughness. Nunner defined the resistance coefficient in terms of the difference between the conventional pressure drop and the work due to acceleration; the latter

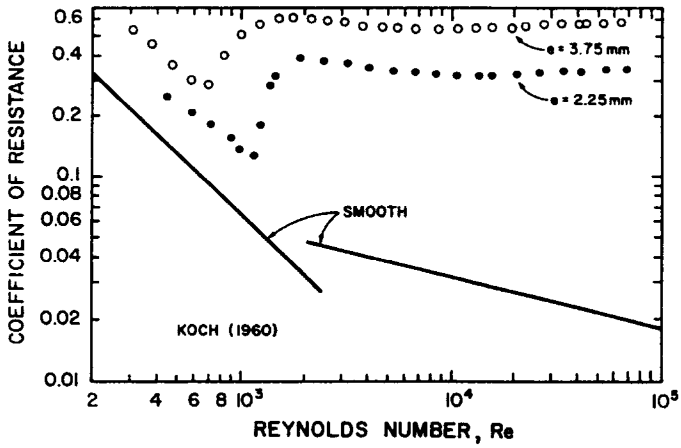

FIG. 2. Effect of e/D on resistance coefficient according to Koch [10].

being expressed as a function of both the pressure and temperature at the inlet and exit of the test section. Obviously, even for isothermal conditions, with nearly the same temperature at the inlet and exit, the latter term always enters into the calculation since the inlet and exit pressures are unequal. Without data for pressure and temperature at the two end points for each trial, the usual friction factor cannot be established from his data. Also, the characteristic length in the Reynolds and Nusselt number was the mean diameter d_m, defined as the equivalent diameter of a smooth pipe having the same volume per unit length. Using the d_m values in his paper (cf. Table 2), all of his results were re-computed using the internal diameter as reference. Even with this revision, the critical Re values are still considerably higher than would be expected for the test conditions. By contrast, in the subsequent study of Koch, the usual definition of friction factor was used and the length parameter was the tube diameter. Suffice it to state that using the empirical relation provided by Koch, it has been estimated that $Re_{c,a} = 1700$ represents the upper limit to the range of critical Reynolds numbers for Nunner's test conditions. The results of other investigations (Table 1) definitely support this view.

A study of the results in Table 1 and Fig. 2 reveals a number of important trends. The most obvious of these is that the critical friction factors are considerably higher, while the critical Reynolds numbers are substantially lower, than the values for smooth tubes. In other words, $\psi_f < 1$ and $\psi_R > 1$ when the data are reduced using the internal diameter. In this regard, it is worthy of note that, if the hydraulic diameter D_e were used as the linear parameter, the critical friction factor can be lower than the smooth tube value, and the values for $Re_{c,a}$ would be even lower than those in Table 1. At this point it is useful to recall that for smooth passages and with D_e (i.e. $\psi_f \simeq 1$), $\psi_R < 1$ for rectangular or annular tubes, but $\psi_R > 1$ for triangular passages.

Another important trend is that $f_{c,a}$ increases, while $Re_{c,a}$ decreases, markedly with increasing height of roughness, regardless of whether one is dealing with transverse repeated-rib inserts, fins, or spirally ridged surfaces. From these results it appears that the roughness height is by far the single most important parameter, at least insofar as the influence on transition is concerned, with moderate effect of pitch. Given the available data base, the fact that the effects of all relevant parameters cannot be established definitely is of little consequence to the present considerations. It is, however, correct to state that, regardless of the geometric configuration of roughness or the prevailing local flow conditions (i.e. separation and reattachment, swirl, etc.), the global effect of roughness is to bring about early transition.

Having discussed the influence of roughness on the critical parameters for transition, it is necessary to consider the fundamental requirement for flow similarity which, as noted earlier, asserts that the critical parameters must be the same. On this basis, it is clear that the three flows of Fig. 2 are not similar, at least not for the $Re > 700$ region, because the onset of transition occurs at about $Re = 702$, 1170 and 2944 for $e = 3.75$, 2.25 and 0 mm (smooth), successively. To obtain realistic information on the physical situation, it is necessary to reduce the data using equations (1) and (2). Accordingly, the original results of Koch [10] and Nunner [9], which are the most extensive, have been re-analyzed, and these are shown graphically in Figs. 3 and 4, respectively, where, in each case, the reduced friction factor f_m is plotted against the reduced Reynolds number Re_m. The fact that Nunner's definition of the resistance coefficient is different, and the somewhat higher $Re_{c,a}$ values from that study, do not preclude this general treatment of his data. Since friction factor is essentially independent of Re in the fully rough regime, these discrepancies are not expected to have a marked effect on the qualitative interpretation of Nunner's results.

Figures 3 and 4 show that, when reduced according to the law of corresponding states, f_m in the laminar

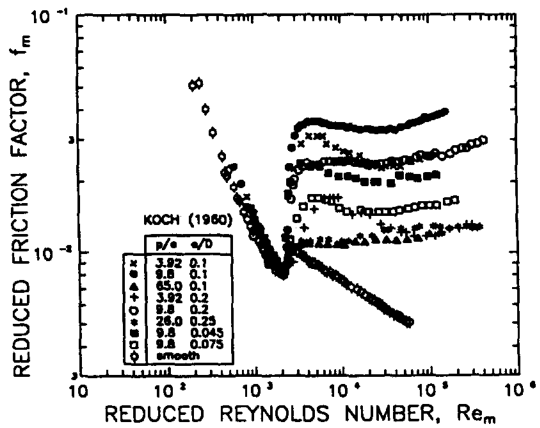

FIG. 3. Similarity plot of f_m vs Re_m using data of Koch [10].

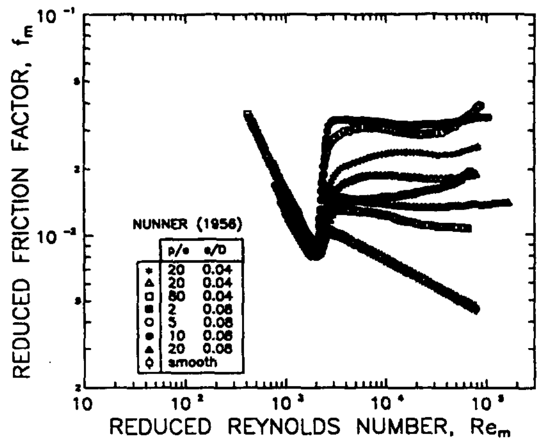

FIG. 4. Plot of f_m vs Re_m using data of Nunner [9].

region is almost independent of the geometric details of the roughness elements and its profile with increasing Re_m is approximated by the smooth tube relation. In the turbulent regime, it is especially noticeable that, relative to the smooth tube data, the absolute increases in f_m are considerably smaller than can be inferred from the conventional f vs Re plot. This fully demonstrates that the marked enhancement factors, as is widely documented in the literature, are intimately associated with transition. For example, whereas Fig. 2 gives enhancement factors of about 17.5 and 20 for $e = 2.25$ and 3.75 mm and $Re = 50\,000$, the corresponding $f_m/f_{m,s}$ $(=\phi_f)$ values from Fig. 3 are about 5.8 and 4.5, respectively. In fact, on Figs. 3 and 4, the largest value for ϕ_f is no more than 10 at $Re_m = 10^5$. And, further, it is quite evident that f_m depends solely on the geometric parameters of the roughness.

It should be noted that the $p/e = 9.8$ results for $e/D = 0.045$ and 0.075 (Fig. 3) are for the situation where a flow path (2.7 mm in width) was provided between the external circumference of the ring insert and the tube wall, while for $p/e = 9.8$ and $e/D = 0.1$, the disc-shaped inserts were rigidly supported on the inner surface of the tube with no clearance. Also, in the case of Nunner's results (Fig. 4), the shape of the roughness elements and the number of such elements in a tube were additional parameters. For instance, with $e/D = 0.08$, the number of rings for $p/e = 10$ was double that for $p/e = 20$, while that for $p/e = 20$ was four times that for $p/e = 80$ with $e/D = 0.04$. For these reasons, it would be unwise to draw any definite conclusion concerning the trend with increasing e/D or p/e from Figs. 3 and 4.

From Fig. 3 or 4 the striking evidence is that the onset of the so-called fully rough regime is almost definite in character and in value, beginning right after the end of the transition region. In this regard the general features of these curves are remarkably similar to those presented by Nikuradse [14] for the largest heights of sand-grain roughness (Fig. 5). In fact, even for the relatively small heights of roughness of that study, where the various curves progressively leave the smooth profile and where the points of successive departure vary with roughness height, it appears that the fully rough regime begins almost at the points at which the individual curves leave the smooth profile.† Although many of the curves exhibit maxima and minima, in line with some of the profiles reported by Nikuradse, the depressions are such that, in each case, the error around the mean value is small. The only exception to this consistent trend can be seen in Fig. 3 for $p/e = 3.92$ with $e/D = 0.1$ and 0.2. These curves are characterized by more pronounced maxima and minima. No definite statement can be made at this point about these trends.

The assertion that different types of roughness exhibit the same general features is strikingly illustrated in Fig. 5, which is a comparison of the reduced results of Fig. 2 with the data of Nikuradse [14]. Given the structural differences between sand-grain roughness and repeated-rib inserts as well as the attendant effects on transition, it would be unwise to draw definite conclusions concerning the modest agreement that is evident from this figure in the turbulent regime.

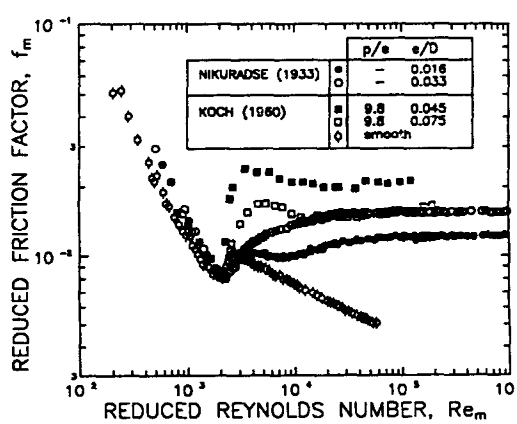

FIG. 5. Nikuradse's sand grain vs Koch's ring inserts.

† See for example, p. 617 of ref. [29].

Nevertheless, the comparison is important in two respects. First, it provides clearer insight as well as possible explanation as to why it has been possible to correlate experimental data using the friction similarity concept advanced by Nikuradse. Second, it supports an observation that the law of resistance in the fully rough regime should be of a universal nature for all pipes [6]. Here, the relevant feature is the dependence of f_m only on the geometric details of roughness, a trend that must also be expected for non-circular passages [12].

3. HEAT TRANSFER

Prior to the presentation and discussion of the heat transfer data, there are two important comments. First, for each of the studies used in the following analysis, the bases for the comparisons and conclusions are the smooth duct data for the particular study. Since the original smooth tube data for some of the studies were not approximated by the Dittus–Boelter or Petukhov–Popov correlation, it can be readily appreciated that the reduced data may not follow either correlation. Second, in the absence of the appropriate temperatures for these studies, no attempt was made to incorporate the Prandtl number effect.

3.1. *Smooth passages*

The role of transition in determining turbulent heat transfer in passages is best illustrated using the modified empirical Filonenko correlation, equation (3), and the Petukhov–Popov correlation [15], equation (4)

$$f_m = (1.58 \ln \psi_R \, Re_a - 3.28)^{-2} \qquad (3)$$

$$Nu_m = (f_m \psi_R \, Re_a \, Pr/2)[1.07 + 12.7(f_m/2)^{1/2}(Pr^{2/3}-1)]^{-1.0}. \qquad (4)$$

Equations (3) and (4) are valid for circular and non-circular passages and, in the case of friction, the alternative form of equation (3) is verified in Fig. 1. It must be noted as a matter of particular importance that, by the introduction of f_m and $Re_m = \psi_R \, Re_a$ into the well-known Petukhov–Popov correlation, the fundamental requirement for similarity for all passages is satisfied.

For rectangular passages, it is especially noticeable that Nu_m decreases with increasing aspect ratio for a given Re based on D_e, as reported by several investigators [16, 17]. Likewise, for tubes of annular section which are characterized by $\psi_R < 1$, the Nu_m trend with increasing diameter ratio (β) is the same as noted above for rectangular channels, and this is definitely confirmed by the results of other researchers [18, 19]. It is also quite apparent that as β and a_r approach zero and unity, respectively, the Nu_m values approach those for circular tubes, as might have been inferred from the general trend for ψ_R with increasing β or a_r [6].

Koch and Feind [19] reported data for heat transfer and frictional pressure coefficients for $0.212 \leqslant \beta \leqslant 0.838$. The results for the latter were thoroughly discussed in ref. [6]. For non-dimensional heat transfer coefficients, laminar and turbulent data exhibited marked dependence on β, decreasing with increasing β. Their results have been re-analyzed and the Nu_m vs Re_m plots are given in Fig. 6. To obtain the corrections for the data of Fig. 6(a), the critical Reynolds numbers given in the aforementioned paper (cf. Table 4) were used along with the limiting circular tube $Re_{c,r}$ value of 2100. For the lower plot (Fig. 6(b)), $Re_{c,r} = 2719$, and this was deduced from the original data of the authors [19].

For the upper and lower plots, although the results with and without the central core are almost indistinguishable, it may be noted that the upper data set are about 25% higher than, while the lower are closely approximated by, equation (4). Figure 6(b) indicates that, to calculate heat transfer coefficients for ducts of arbitrary shapes, equation (4) can give satisfactory results provided the appropriate critical parameters (ψ_f, ψ_R) are used to account for the role of transition. The closeness with which the data for all β are approximated by a single regression line also supports the view that heat transfer coefficients decrease with increasing $Re_{c,a}$. The trend on Fig. 6(a) also establishes the importance of determining the $Re_{c,r}$ value for any particular duct geometry.

For triangular passages, $\psi_R > 1$ and Re_m increases with increasing h/b (or with decreasing apex angle in

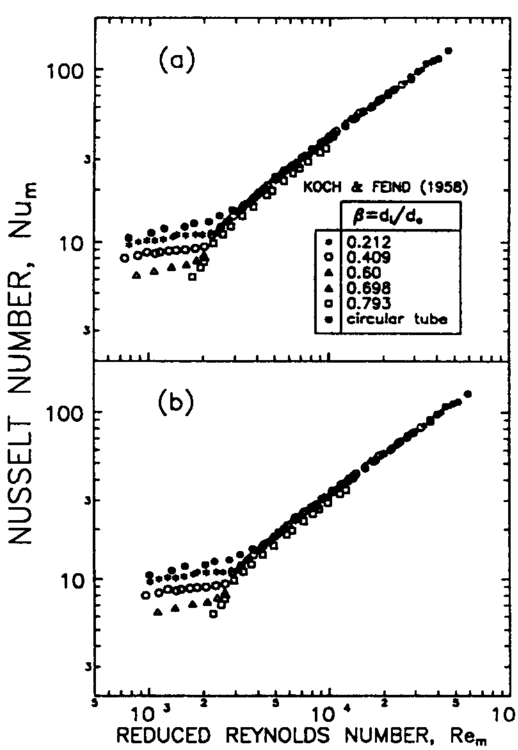

FIG. 6. Nusselt number (Nu_m) vs Re_m for smooth annuli.

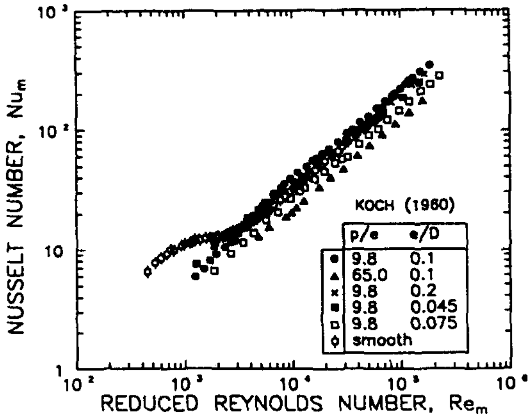

FIG. 7. Nusselt number (Nu_m) vs Re_m using data of Koch [10].

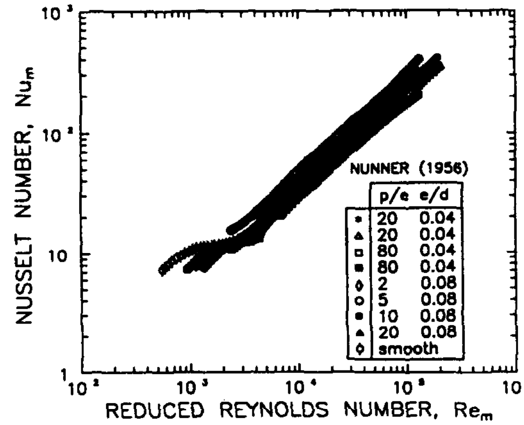

FIG. 8. Nu_m vs Re_m using data of Nunner [9].

the case of isosceles ducts) for a given Reynolds number based on D_e. Consequently, Nu_m values can be substantially greater than for circular, rectangular or annular geometry with the same Reynolds number. For instance, using the α (apex angle) = 38.8° data in ref. [6] which gives $\psi_R = 1.197$ and for $Re = 80\,000$ (i.e. $Re_m = 95\,760$), Nu_m is about 163, which is about 15% higher than with a circular tube. In fact, for the same Re, the calculated value for the narrowest, $\alpha = 4.01°$ passage ($Nu_m = 196$) is about 40 and 60% higher than for a cylindrical and rectangular ($a_r = 10$) geometry, respectively. The common basis for these calculations is $Pr = 1$. This observation of great practical importance, though clearly at variance with the heat transfer results of Eckert and Irvine [20] or Tokarev [21], is not altogether surprising for several reasons. In the first place, to maintain a fixed Reynolds number the average flow velocity must rise with increasing h/b, due to the reduction in flow area. Further, the critical Re attains its lowest value for these narrow passages [6] and, even at such relatively low Re, the coexistence of laminar and turbulent flow [22, 23] should cause a marked improvement in heat transfer over that for completely laminar flow.

3.2. *Rough passages*

In order to gain clearer insight into the influence of transition on heat transfer, some of the results of Koch [10] for which the critical Reynolds numbers were determined precisely have been re-analyzed and these are shown on Fig. 7 where Nu_m is plotted against the reduced Reynolds number, Re_m. It should be emphasized that as with Fig. 6, the corrections have been applied only to the familiar Reynolds numbers. The $e = 2.25$ and 3.75 mm data, for which the friction results were given in Fig. 2, correspond to those for $e/D = 0.045$ and 0.075, respectively, and these are almost coincident with the smooth tube data. The fact that some of the reduced data lie moderately above and below the smooth tube data does not matter, since the transition parameter (ψ_R) is based on isothermal friction data. Since ψ_R varies according to whether the fluid is being heated or cooled and with the temperature difference between the surface and the fluid [24], it must be expected that whatever might be the effects of heat transfer on ψ_R, these must be more pronounced for rough than for smooth passages.

Figures 8 and 9 which are alternative representations of the results of Nunner [9] and Blackwelder and Kreith [25] complement one another in that, unlike the use of ψ_R values deduced from isothermal friction factor data, the ψ_R values for these results were determined from the heat transfer data. In principle, this would be one of the preferred methods for determining ψ_R because it eliminates its dependence on heat transfer (heating or cooling) and temperature. The difficulty, however, is that when $Re_{c,a}$ is unusually low, as was the case with some of Koch's test conditions, it becomes almost impracticable to obtain accurate heat transfer coefficients at such low flow rates. Whereas Nunner used repeated-rib inserts (Table 1), Blackwelder and Kreith tested twisted tape inserts and the relative lengths (p_t/D), where p_t is the length required for a 360° rotation of the tape, are given on Fig. 9. In each case, it may be noted that virtually all of the reduced data lie in a band about ±15% around the smooth tube data, fully supporting

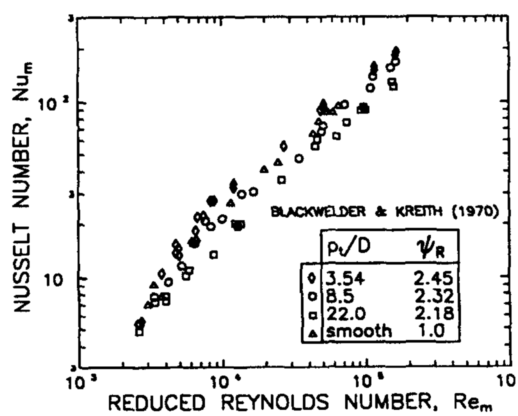

FIG. 9. Nu_m vs Re_m using data of Blackwelder and Kreith [25].

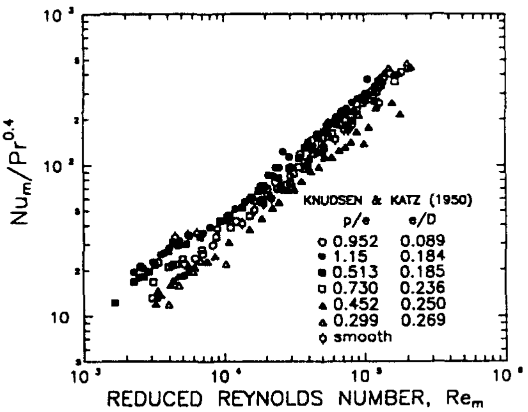

FIG. 10. ($Nu_m/Pr^{0.4}$) vs Re_m using data of Knudsen and Katz [12] for annuli.

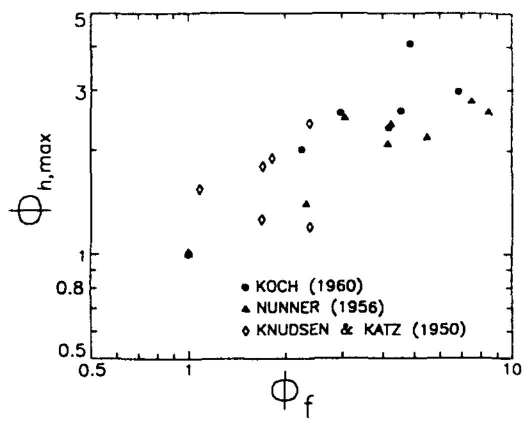

FIG. 11. Variation of $\phi_{h,max}(=(Nu_r/Nu_s)_{max})$ with $\phi_f(=f_m/f_{m,s})$.

the view that good estimates of average heat transfer coefficients can be made provided values of the critical *Re* for transition are known.

Another study uncovered during the exhaustive literature search, that by Knudsen and Katz [12], dealt with heat transfer and pressure drop in annuli with transverse fins. The significance of this study relates to the use of water as the working fluid, in contradistinction to the air studies of Figs. 7–9. Another distinction between the air and water studies relates to the use of annular passages instead of circular tubes, inasmuch as the analysis should provide at least partial verification of the law of corresponding states for rough passages of arbitrary shapes.

Since the smooth duct relative entrance length (L_e/D_e) of Knudsen and Katz was 31.0, not sufficiently long to give fully developed flow for their well-rounded entrance, and since the critical *Re* was about 1770, it was necessary to establish a reference fully developed flow value for a passage of $\beta = 0.278$. Using isothermal friction factor results presented in Fig. 7 of their paper, this reference $Re_{c,r}$ was estimated to be 2989, and it corresponds to the value at which the critical friction factor attains the smooth duct value of 0.008. It may be noted that the above $Re_{c,r}$ value is not significantly different from that ($Re_{c,r} = 2873$) determined from the results of Koch and Feind [19] for $\beta = 0.212$ and $L_e/D_e = 50$ [6]. The reduced data for the smooth and finned tubes are presented in Fig. 10. In the absence of the *Pr* value for each test trial, the ordinate ($Nu_m/Pr^{0.4}$) is the same as in the original paper. Again, the outcome of the analysis is essentially the same as charted already in Figs. 7–9, with about a $\pm 20\%$ spread around the authors' smooth tube data.

It is quite evident that, in each of Figs. 7–10, most of the reduced data lie in a band about $\pm 15\%$ around a regression line for the smooth tube data. From a careful analysis of the results presented in Koch [10] for a range of conditions, it appears that the reduced data may lie well below the smooth data for certain values of $Re_{c,a}$. Whether there exists a definite $Re_{c,a}$ range below which the Nu_m data would deviate markedly from those for smooth tubes, or how far down the Reynolds number scale such a value may occur, is difficult to ascertain from the results available. Suffice it to state that the existence of such a limiting critical Reynolds number would not in any way weaken the general conclusion regarding the decisive role of transition. Bearing in mind that the ratio Nu_m/Nu_s will always vary from slightly above to below unity, such information may be useful in the assessment of the thermal efficiency of various roughness geometries.

In summary, the general trends on Figs. 7–10 imply several things. First, they indicate that the increases in heat transfer coefficients may be closely associated with transition from laminar to turbulent flow. Second, they suggest that heat transfer coefficients for rough passages can be estimated from the smooth tube data provided the critical Reynolds numbers are known. Finally, it appears that the familiar Reynolds number (*Re*) dependence in the turbulent regime is not markedly different from that for smooth tubes.

To shed further light on the connection between transition, friction and heat transfer, the results of several investigators are presented differently on Fig. 11 where $\phi_{h,max}$ ($= Nu_r/Nu_s)_{max}$ is plotted against ϕ_f ($=f_m/f_{m,s}$). Naturally, since the smooth duct friction factor ($f_{m,s}$) decreases with Re_m for turbulent flow, with a nearly constant f_m for a given set of roughness geometry, an arbitrary value of $Re_m = 50\,000$ was used for calculations of ϕ_f. In going from $Re_m = 50\,000$ to $100\,000$, $f_{m,s}$ changes by no more than 20%; hence the use of the latter Re_m value does not modify the trend on this figure. Since $Nu_m/Nu_s \rightarrow 1$ (Figs. 7–10), it was considered more practical to define $\phi_{h,max}$ using Nu_r and Nu_s.

Figure 11 shows some scatter, but nothing that is unexpected considering the differences between the three studies, the lack of geometrically similar test conditions, the dependence of ψ_f and ψ_R on heat transfer and surface temperature and the difficulties associated with accurate determination of the effect of duct

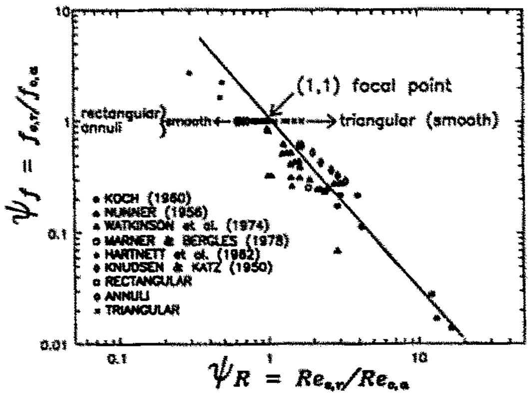

FIG. 12. Generalized plot of ψ_f vs ψ_R for smooth and rough passages.

geometry on transition. It is nevertheless a fact of considerable importance, that $\phi_{h,max}$ attains a value of no more than 5 for $1 < \phi_f < 9$. As to whether there should be a unique relation between f_m and Nu_m or between $\phi_{h,max}$ and ϕ_f; although no relation of a general nature can be established from Fig. 11, the trend on this figure clearly suggests that a definite relation might result from systematic measurements.

4. DISCUSSION

The numerous observations on the effects of transition on heat transfer coefficients for smooth and rough passages can best be integrated with the help of Fig. 12, which is a plot of $\psi_f = (f_{c,r}/f_{c,a})$ vs $\psi_R = (Re_{c,r}/Re_{c,a})$ using the data of Table 1. A few results for smooth rectangular, annular and triangular passages from ref. [6] and the three data points obtained by Hartnett *et al.* [26] with a smooth entrance are also included in Fig. 12. Since ψ_f (i.e. $f_{c,r}/f_{c,a}$) is nearly unity for smooth passages [6], these data are located along $\psi_f = 1$. For clarity, the data have not been identified according to the aspect, diameter or height-to-base ratio. It is important to note, however, that the focal point on this figure is $(\psi_f, \psi_R) = (1,1)$ and this corresponds to fully developed flow in a circular tube with a limiting $Re_{c,r} = 2100$. The aspect (or diameter) ratio of the smooth passage is increasing with decreasing ψ_R, while the triangular duct height-to-base ratio is increasing monotonically with ψ_R, from this focal point.

In terms of (ψ_f, ψ_R), the parametric description of the heat transfer process amounts to this: heat transfer coefficients are higher than smooth circular tubes for $\psi_f \leqslant 1$ and $\psi_R > 1$, while the trend is exactly the opposite for $\psi_f \geqslant 1$ and $\psi_R < 1$. It is especially noticeable that smooth triangular passages produce an effect on transition ($\psi_R > 1$) which is similar to that due to roughness, fully supporting the conclusions in Section 3.1 on the smooth duct heat transfer trends. In this paper, the situations: $\psi_f \leqslant 1$, $\psi_R \geqslant 1$ and $\psi_f = 1$, $\psi_R < 1$ have been addressed satisfactorily. The condition, $\psi_f > 1$ and $\psi_R < 1$, is a special case and this corresponds to the situation where special precautions are taken to delay transition. No improvements in heat transfer should be expected for $\psi_f > 1$ and $\psi_R < 1$, and this has been confirmed by numerous results [27, 28]. We are therefore led to the following conclusion: to alter the existing heat transfer for a given set of conditions, be it in the direction of higher or lower rates, it may be necessary to change the critical Reynolds number or critical friction factor for transition. In passing, it is of interest to note that the data of Hartnett *et al.* for a very smooth entrance do not lie far from the extrapolated regression line obtained using the data of Koch [10], suggesting that, if systematic measurements were made, a relation between ψ_f and ψ_R may exist for all situations where the onset of transition is effected artificially.

In general, the conditions of the local flow over roughness elements depend, to a large extent, only on the geometric details of the elements. For transverse protrusions or grooves, the dominant features are separation and reattachment, while visual studies have confirmed the existence of flow rotation and separation for spirally-ridged surfaces. From much that has been presented so far, it appears that the geometric parameters of roughness, in determining the prevailing local flow conditions, also produce a global effect on transition, in a manner that is consistent with the well established influence of single and two-dimensional roughness in incompressible boundary layer flows [29].

5. CONCLUSIONS

The determination of the role of transition on friction and convective heat transfer, which was the objective of this analysis, has been accomplished satisfactorily. The most important observations are as follows.

(1) Transition can have a marked effect on the attainable friction and heat transfer coefficients in smooth and rough passages.

(2) According to the frictional law of corresponding states, different types of roughness exhibit the same general features at the same reduced conditions. The onset of the fully rough flow regime is almost definite in value and character, beginning right after the transition region.

(3) For rough passages, the marked increases in frictional pressure coefficient are intimately associated with early transition and, when this role of transition is accounted for, the reduced friction factors are considerably lower than the values deduced from the conventional f vs Re plot.

(4) For heat transfer, there are indications that the lower the critical Reynolds number for transition, the greater the average heat transfer coefficients, regardless of whether the passage is smooth or rough.

(5) For smooth passages, average heat transfer coefficients may be greatest with triangular passages, lowest with ducts of annular or rectangular geometry,

but intermediate values should be expected for circular tubes.

(6) For rough passages, there are clear indications that estimates of the average heat transfer coefficient, that are quite satisfactory for design purposes, can be obtained from the smooth tube data by replacing the familiar Reynolds number (Re_a) with the reduced value (Re_m) computed from $Re_m = \psi_R\ Re_a$.

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LE ROLE DE LA TRANSITION DANS LA DETERMINATION DU TRANSFERT THERMIQUE ET DU FROTTEMENT DANS DES PASSAGES LISSES ET RUGUEUX

Résumé—La transition peut avoir des effets profonds sur le frottement et le transfert thermique dans les passages d'écoulement. En accord avec la loi proposée des états correspondants pour le frottement, différents types d'éléments de rugosité montrent le même comportement, pour des mêmes conditions réduites de frottement, que dans le cas des passages lisses de forme arbitraire. Pour des tubes lisses et rugueux, il semble que le coefficient moyen de transfert thermique diminue quand le nombre de Reynolds critique de transition augmente. Pour des passages lisses non circulaires, les coefficients moyens de transfert thermique peuvent être calculés à partir des relations du tube circulaire, si les valeurs critiques du coefficient de frottement et du nombre de Reynolds sont connues.

DIE BEDEUTUNG DES ÜBERGANGS BEI DER BESTIMMUNG VON REIBUNG UND WÄRMEÜBERGANG IN GLATTEN UND RAUHEN STRÖMUNGSKANÄLEN

Zusammenfassung—Der Übergang kann einen spürbaren Einfluß auf Reibung und Wärmeübergang in Strömungskanälen haben. Entsprechend dem vorgeschlagenen Gesetz der korrespondierenden Zustände für die Reibung zeigen unterschiedliche Rauhigkeitselemente dasselbe Verhalten im Hinblick auf die Reibung bei denselben reduzierten Zuständen, wie dies auch bei glatten Kanälen beliebiger Form der Fall ist. Für glatte und rauhe Rohre scheint der mittlere Wärmeübergangskoeffizient mit steigender kritischer Reynolds-Zahl im Übergangsbereich abzunehmen. Für glatte Strömungskanäle, die nicht kreisrunden Querschnitt besitzen, kann der mittlere Wärmeübergangskoeffizient aus Berechnungen für das kreisrunde Rohr ermittelt werden, und zwar für den Fall, daß die kritischen Werte für den Reibungsbeiwert und die Reynolds-Zahl bekannt sind.

РОЛЬ УСЛОВИЙ ПЕРЕХОДА ПРИ ОПРЕДЕЛЕНИИ КОЭФФИЦИЕНТОВ ТРЕНИЯ И ТЕПЛОПЕРЕНОСА В КАНАЛАХ С ГЛАДКИМИ И ШЕРОХОВАТЫМИ СТЕНКАМИ

Аннотация—Условия перехода ламинарного течения в турбулентное оказывают большое влияние на трение и теплоперенос в каналах. Согласно предложенному закону соответственных состояний для трения различные типы элементов шероховатости в каналах оказывают такое же влияние на трение при идентичных приведенных условиях, как и в случае гладких каналов произвольной формы. Для гладких и шероховатых каналов оказывается, что средний коэффициент теплопереноса уменьшается с увеличением критического значения числа Рейнольдса. В случае некруглых гладких каналов средний коэффициент теплопереноса можно рассчитать по соотношениям для круглых труб при условии, что критические значения коэффициента трения и числа Рейнольдса известны.